Quantification, witness sets and conservativity

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An important cross-linguistic observation about the meaning of natural language determiners, first pointed out by Barwise and Cooper (1981) and Keenan and Stavi (1986), is that the truth of an expression of the form \([\text{DET} + \text{N}] + \text{V}\) never depends on the set \([\text{V}] - [\text{N}]\). Within Generalised Quantifier Theory, where determiners like e.g., every, most, no, at most three are analysed as denoting functions \(f\) from subsets of the domain \(D\) of entities to functions from subsets to truth-values (or equivalently as binary relations between subsets of \(D\)), this observation has been expressed by saying that these functions \(f\) satisfy the property of conservativity, i.e., \(f(A)(B) = 1 \iff f(A)(A \cap B) = 1\). But crucially, conservativity is not derived as a consequence of GQ Theory. The aim of this paper is to sketch the basic idea behind a new theory of quantification which, unlike GQ Theory, derives (and thus explains) the fact that the truth of an expression of the form \([\text{DET} + \text{N}] + \text{V}\) never depends on the set \([\text{V}] - [\text{N}]\). To do so we shall (have to) break with tradition in several ways.

First, I assume that determiners denote unary functions from subsets \(R\) of \(D\) into pairs \(\langle R, W \rangle\), where \(W\) is the set of subsets \(X\) of \(R\) which satisfy the condition imposed by the determiner. We shall refer to \(X\) as a witness set, and to \(W\) as the set of witness sets. For example, the determiner exactly two denotes the function \(f\) such that for any \(R \subseteq D\), \(f(R) = \langle R, \{ X : X \subseteq R \land |X| = 2 \} \rangle\). To illustrate, in a model with \(D = \{ a, b, c, d, e, f, g, h \}\), \([\text{professor}] = \{ a, b, c \}\) and \([\text{student}] = \{ d, e, f, g \}\), applying the denotation \([\text{exactly two}]\) to the denotation \([\text{professor}]\) results in \([\text{exactly two}]([\text{exactly two}]) = \{ \{ a, b, c \}, \{ \{ a, b \}, \{ a, c \}, \{ b, c \} \} \}\), while \([\text{no}]([\text{exactly two}]) = \{ \{ a, b, c \}, \{ \} \}\).

Secondly, ignoring tense, aspect and events for simplicity of exposition) verbs are taken to denote pairs \([P, S]\) consisting of a relation \(P \subseteq D^n\) (for some integer \(n > 0\)) and a store \(S\) containing the information about the semantic role assignment to DP denotations (for lexical verbs the store is the empty set).

Thirdly, instead of using type-driven semantic composition, a semantic operation (schema) \(\mathcal{O}_i\) is proposed, which adds the pair \([i, [DP]]\) to the store of \([V]\), so that \(\mathcal{O}_i([DP]), (P, S) = (P, S \cup \{ [i, [DP]] \})\), if there is no \(X\) with \(i, X \in S\). To illustrate, let \([\text{examined}] = \langle E, \emptyset \rangle\), with \(E = \{ \{ a, d \}, \{ a, c \}, \{ a, f \}, \{ b, d \}, \{ b, c \}, \{ b, g \}, \{ h, d \}, \{ h, e \}, \{ h, g \} \}\). The underspecified denotation of the sentence Exactly two professors examined exactly three students, is:

\[
\begin{align*}
\mathcal{O}_2([\text{exactly three students}], \mathcal{O}_1([\text{exactly two professors}, [\text{examined}]]) & = \\
\mathcal{O}_2([\text{exactly two professors}], \mathcal{O}_2([\text{exactly three students}, [\text{examined}]]) & = \\
\mathcal{O}_1([\text{exactly two professors}], \mathcal{O}_2([\text{exactly three students}, [\text{examined}]]) & = \\
\langle E, \{ \{ i, \text{exactly two professors} \}, \{ 2, \text{exactly three students} \} \rangle & = \\
\end{align*}
\]

where the assignment of semantic roles is fixed, but the scope dependencies are left unspecified. An \(n\)-ary formula \(\phi = \langle V, S \rangle\) is true relative to a sequence of expansions \(\langle \alpha_1, \ldots, \alpha_m \rangle, 0 \leq m \text{ iff there are DP denotations } X_1, \ldots, X_n \text{ such that (i) } \langle i, X_i \rangle \in S \text{ for all } 1 \leq i \leq n, \text{ (ii) } \langle X_1, \ldots, X_n \rangle \in \pi_1(\alpha_1(\alpha_2(\ldots \alpha_m(\phi))))\).

Fourthly, both the readings with dependent (e.g., direct, inverse) as well as those with independent (e.g., cumulative) scope dependencies are derived by applying expansion operations to the same underspecified denotation (1). Sequential \(i\)-expansion accounts for the direct and inverse scope readings, whereas simultaneous expansion accounts for the cumulative reading. The sequential expansion operation is an adaptation of the expansion operation presented in Akiba (2009). To illustrate the basic idea of expansion, if sequential 2-expansion is applied to (1), the pair \([a, \text{exactly three students}]\) is added to \(E\), because professor \(a\) stands in the examine relation to exactly three students, namely \(d, e\) and \(f\). Given that the professors \(b\) and \(h\) also stand in the examine relation to exactly three students, namely \(d, e, g\), the expansion of the second projection also adds the pairs \([b, \text{exactly three students}]\) and \([h, \text{exactly three students}]\).

Since no other professor stands in the examine relation to exactly three students, no more pairs are added to \(E\). Applying next sequential 1-expansion to the resulting denotation \(\langle E', S \rangle\), (only) the pair \([\text{exactly two professors}]\), \([\text{exactly three students}]\) is added to \(E'\), because \(a\) and \(b\)
are the only professors who stand in the examine relation to exactly three students.

(2) **Definition (sequential $i$-expansion):**
Let $[\phi] = \langle V, S \rangle$ be the denotation of an $n$-ary formula with $\langle j, (R_j, W_j) \rangle \in S$ for all $j$, $1 \leq j \leq n$, and let $\sigma[i/x]$ be the sequence resulting from replacing the element in the $i$-th position of $\sigma$ by $x$. Then $\text{EXP}^{\text{SEQ}}_i([\phi]) = \langle V', S - \{\langle i, (R_i, W_i) \rangle\} \rangle$, where $V'$ is the smallest set satisfying the conditions that (i) $V \subseteq V'$ and (ii) $\sigma[i/(R_i, W_i)] \in V'$ if there is a witness sets $X \in W_i$ and a sequence $\sigma \in V$ such that $\forall x \in R_i, \langle x, x \rangle \leftrightarrow \sigma[i/x] \in V'$.

The cumulative reading can be derived (using the same DP denotations) by applying the simultaneous expansion operator. To illustrate consider *Exactly two professors criticised exactly three students*, where $\text{criticised} = \{\langle a, d \rangle, \langle a, e \rangle, \langle b, e \rangle, \langle b, f \rangle\}$.

(3) **Definition (simultaneous expansion):**
Let $[\phi] = \langle V, S \rangle$ be the denotation of an $n$-ary formula with $\langle j, (R_j, W_j) \rangle \in S$ for all $j$, $1 \leq j \leq n$. Then $\text{EXP}^{\text{SIM}}([\phi]) = \langle V', \emptyset \rangle$, where $V'$ is the smallest set satisfying the conditions that (i) $V \subseteq V'$, and (ii) $\langle (R_1, W_1), \ldots, (R_n, W_n) \rangle \in \text{EXP}^{\text{SIM}}([\phi])$ if there are $X_1 \in W_1, \ldots, X_n \in W_n$ such that $\forall x \in R_1 \ldots \forall z \in R_n (\langle x, \ldots, z \rangle \in X_1 \times \ldots \times X_n \leftrightarrow \langle x, \ldots, z \rangle \in V)$

Note that the two witness sets $\{a, b\}$ and $\{d, e, f\}$ of [exactly two professors] and [exactly three students] satisfy the second condition.

Finally, some important differences between this proposal and other quantification theories are discussed. First, unlike e.g. in GQ theory, here we can actually derive that the truth of DET+N+V sentences never depends on $[V] \setminus [N]$. Second, unlike the verb denotations proposed in Cooper (1983) or Hendriks (1993), Kobele (2006) which result from type-lifting to the worst case, the verb denotations here are not blindly expanded to the worst case (as e.g. in Akiba (2009)), but only relative to the respective restrictors. And finally, unlike in many approaches to underspecification, the underspecified denotations here are not sets of all available readings, but the common denominator of all readings in the sense that the expanded relations $V'$ are supersets of the underspecified relations $V$.

**References**


